

The Cosmological Constant From The Viewpoint Of String Theory

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The mystery of the cosmological constant is probably the most pressing obstacle to significantly improving the models of elementary particle physics derived from string theory. The problem arises because in the standard framework of low energy physics, there appears to be no natural explanation for vanishing or extreme smallness of the vacuum energy, while on the other hand it is very difficult to modify this framework in a sensible way. In seeking to resolve this problem, one naturally wonders if the real world can somehow be interpreted in terms of a vacuum state with unbroken supersymmetry.

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The problem of the vacuum energy density or cosmological constant – why it is zero or extremely small by particle physics standards – really only arises in the presence of gravity, since without gravity, we don't care about the energy of the vacuum. Moreover, it is mainly a question about quantum gravity, since classically it would be more or less natural to just decide – as Einstein did – that we do not like the cosmological constant, and set it to zero.

Classically, it may involve some fine-tuning to set the cosmological constant to zero, but once this is done, that is the end of the story. Quantum mechanically, it will not help to just set the cosmological constant to zero in a microscopic Lagrangian. The observed value of the vacuum energy density potentially includes contributions from things such as loops of soft photons. The puzzle is why the vacuum energy is so small after including all of these contributions.

As the problem really involves quantum gravity, string theory is the only framework for addressing it, at least with our present state of knowledge. Moreover, in string theory, the question is very sharply posed, as there is no dimensionless parameter. Assuming that the dynamics gives a unique answer for the vacuum, there will be a unique prediction for the cosmological constant.

But that is, at best, a futuristic way of putting things. We are not anywhere near, in practice, to understanding how there would be a unique solution for the dynamics. In fact, with what we presently know, it seems almost impossible for this to be true, rather as a few years ago it seemed almost impossible that the different string theories would turn out – as they have – to be limiting cases of one more unified theory.

Not understanding why the cosmological constant is zero, or extremely small, is in my opinion the key obstacle to making the models of particle physics that can be derived from string theory more realistic. In fact, as I will explain, that has been true since 1985.

Any time that one does not understand something, one can point to details that do not work. It is always important to identify what is wrong qualitatively and gives the best clue to possible future progress.

Let me give an analogy. In the early 1980's, it was fairly clear that string theory gave consistent models of quantum gravity and that in that framework, one was forced to unify gravity with other forces. Many details were not right, but the most striking qualitative problem was that in the models of particle physics that could be derived from string theory at that time *the weak interactions would have to conserve parity*. Parity violation in weak interactions is deeply embedded in the standard model, and the inability to reproduce it

is a much more basic problem than an inability to compute the correct mass of the muon, for example.

When this problem was cleared up, by the Green-Schwarz generalized anomaly cancellation mechanism in 1984, many other things fell into place – including the discovery of the heterotic string shortly afterwards – and the particle physics models became much more realistic.

Since that time, the outstanding qualitative problem has been the cosmological constant, or more exactly, why it is zero or extremely small after supersymmetry breaking. Broadly speaking, models of particle physics derived from string theory work fine in the absence of supersymmetry breaking; but we do not have a convincing method of supersymmetry breaking. Hence, we can give a reasonably elegant (though not rigorous) explanation of quark and lepton gauge interactions and family structure. But we cannot say very much about masses or mixing angles.

The basic diagnostic test for knowing that a wrong mechanism of supersymmetry breaking is wrong is that it generates a cosmological constant. On this basis, all the known approaches to supersymmetry breaking are wrong. Thus, the cosmological constant is the main clue to improving the particle physics models.

Without supersymmetry breaking, there is no problem in getting a stable vacuum with zero cosmological constant. If ϕ is a generic scalar field and $V(\phi)$ is the effective potential, then with unbroken supersymmetry one can naturally have a stable minimum of $V(\phi)$ at which $V = 0$. (In the abstract, unbroken supersymmetry allows negative V as well, but when one actually tries to construct string theory models that could reproduce the supersymmetric standard model – for example via Calabi-Yau compactification – one naturally finds $V = 0$.)

Known mechanisms of supersymmetry breaking typically give an unstable runaway, with V positive but vanishing as ϕ goes to infinity, or else they give a V that is not positive definite. (The runaway involves one or more scalars, depending on the model considered. I will use “ ϕ ” as a generic label for such scalars.) This is linked to the fact that the mechanisms of supersymmetry breaking that we understand generally turn off when the string coupling constant goes to zero or a compactification radius goes to infinity. And there generally is a scalar field ϕ that controls the string coupling constant or compactification radius. So supersymmetry breaking turns off as ϕ goes to infinity [1].

Scenarios with an unstable runaway have been considered by many physicists over the years. As far as I know, such scenarios were first considered by Dirac in his approach

to the large numbers problem. In recent years, experimental evidence for a nonvanishing cosmic energy density in today's universe has suggested that (as an alternative to a true cosmological “constant”) the actual universe may be undergoing such a runaway. Such scenarios have been much discussed (along with other possible cosmic components of negative pressure) under the name “quintessence.” A discussion of the observational situation can be found in [2]. These issues will be further addressed by other speakers at the present meeting.

Going back to string theory, in view of the remarks in the last paragraph, supersymmetry breaking mechanisms that lead to an unstable runaway – $V(\phi)$ has its minimum at $\phi = \infty$, where it vanishes – should be considered seriously. But there are severe difficulties with them. First of all, as $\phi \rightarrow \infty$, we need the bose-fermi mass splittings to vanish fast enough relative to the vanishing of $V(\phi)$. This is simply the cosmological constant problem restated for the case of a runaway solution rather than a stable vacuum. Known supersymmetry breaking mechanisms fail this test.

Moreover, even if we can arrange for $V(\phi)$ to vanish fast enough for large ϕ , compared to the mass splittings, this kind of scenario has to face acute obstacles. Unless we restrict the couplings of ϕ in a way that has no apparent rationale in mechanisms of supersymmetry breaking that I can think of, we will find that as ϕ varies, the natural “constants” will change in time. We are at risk of finding \dot{G}/G and $\dot{\alpha}/\alpha$ close to $\dot{\phi}/\phi$ (here G is Newton's constant, measured in units in which \hbar , c , and the proton mass are fixed; α is the fine structure constant). By making V sufficiently flat so that $\dot{\phi}/\phi$ is sufficiently small, and making the couplings of ϕ sufficiently weak, we can perhaps bring \dot{G}/G and $\dot{\alpha}/\alpha$ within acceptable limits.

Even so, we will still, in general, be in trouble because if V is so flat and $\dot{\phi}$ so small as such a scenario will require, then ϕ will behave in laboratory and solar system measurements as a massless field. We will generically expect that coherent and perhaps even spin-dependent forces mediated by ϕ will show up in tests of the equivalence principle, solar system tests of General Relativity, and perhaps in “fifth force” measurements. I actually personally find it very surprising, if nature is exhibiting a runaway with cosmic evolution of a scalar field ϕ , that the effects of the ϕ field have not been seen directly already in one or more of the experiments that I have mentioned.

Especially if experimental indications of a cosmological “constant” hold up, I think that experiments that will improve the bounds on ϕ couplings are important and promising. One proposal, for example, is a satellite measurement (STEP) that could possibly improve

the tests of the equivalence principle by a factor of 10^6 . This would improve the bounds on the couplings of ϕ by a factor of 10^3 . The bound at present is roughly speaking that the couplings of ϕ are a couple orders of magnitude smaller than gravitational coupling. In string theory, a ϕ field typically has a coupling not much weaker than gravity, and suppression of the ϕ couplings relative to gravity puts a severe restriction on the model. From this perspective, in runaway scenarios it would be surprising if ϕ exists and would not be detected in an experiment that would improve the test of the equivalence principle by a factor of 10^6 .

I have emphasized scenarios with a runaway because I think that they are suggested by experimental findings of possible cosmic acceleration. One advantage of a runaway rather than a true cosmological “constant” is that, by analogy with a zero cosmological constant scenario first outlined by Dyson many years ago [3,4], in a runaway scenario, life can possibly adapt and survive and develop forever by working at lower and lower temperatures and with longer and longer time and length scales. A strict cosmological constant would bring all this to a grim end by introducing effective length and time cutoffs. On the other hand, as I have also tried to explain, the difficulties with runaway scenarios are so severe that one would probably have to find something really interesting and new to get a scenario that works.

I should also mention an alternative (which still fits within the general rubric of “quintessence”) which is less ambitious and dramatic than the runaway but faces much less severe experimental difficulties. We could assume that the potential $V(\phi)$ has a stable minimum at which it vanishes, but that the actual universe has not yet reached the minimum because the potential is very flat. For example, ϕ might be an axion-like field, which in particular is angle-valued. The potential, coming from instantons, might be something like $V(\phi) = V_0(1 - \cos \phi)$, with V_0 a constant. I have adjusted an additive constant in V so that the true minimum of the potential (at $\phi = 0$) has $V = 0$ and zero cosmological constant. Either because V_0 is very small or the universe started near the top of the potential bump at $\phi = \pi$ (or both), ϕ might still in today’s universe be away from the minimum of the potential and very slowly changing so that it imitates a cosmological constant. Axion-like fields that get exponentially small potentials of roughly this kind do occur in many string models. If the universe is near the maximum of the potential today, or at least not too close to the minimum, then V_0 should be of order the observed vacuum energy density. *A priori*, this involves some fine-tuning, as does any description of a tiny but nonzero vacuum energy density. Such a scenario is not as challenging conceptually as

a runaway. However, it has the practical advantage that there are mechanisms for suppressing the axion couplings to ordinary matter that would not apply to moduli that might lead to a runaway. Hence, in this kind of scenario, the experimental limits on light scalars are potentially much less problematic. In such a scenario, theorists would need to focus on why the true cosmological constant is exactly zero and why there is an axion with the right value for V_0 .

What about other scenarios? Interestingly, I cannot think of any known assumption or approximation in string theory that leads to the most obvious interpretation of recent experimental findings: a stable vacuum with a positive cosmological constant. There certainly is not an attractive known way to do this, and we certainly don't have the techniques at the moment to find such a vacuum in which, in addition, the cosmological constant is extremely small and the particle physics is attractive.

We do have stable vacua with negative cosmological constant (and sometimes unbroken supersymmetry). Such vacua presumably do not describe the real world, but exploring them has been in the last few years, beginning with Maldacena's proposed duality [5], an arena for amazing theoretical advances. From this work has come new approaches to nonperturbative description of quantum gravity, understanding of quark confinement in terms of properties of black holes, and more.

What I have said so far has been very general. In the concluding portion of this talk, I will make a few remarks on some specific approaches to the cosmological constant problem. First of all, the problem is hard because of the following set of facts:

- (1) Solving the problem seems to require a low energy mechanism – to cancel contributions of loops of soft photons, for example.
- (2) But low energy physics in the standard framework of four-dimensional effective field theory does not seem to offer a solution to the problem. For a review of these issues, see [6].
- (3) It is very hard to change the low energy framework in a sensible way, given all of the familiar successes.

Faced with this conundrum, one way out (eloquently described by Weinberg in the lecture before mine) is an anthropic principle. This entails abandoning the quest for a conventional scientific explanation and interpreting the smallness of the cosmological constant as a necessary feature of our local environment, without which our human species could not have developed to ponder the question. I very much hope that things will not go that way, for several reasons. First, I think we need the cosmological constant as a

clue to understand particle physics better; if the cosmological constant must be treated anthropically – and the approximate vacuum we live in is drawn fortuitously from an ensemble – I am not optimistic about how well we will be able to understand the aspects of particle physics that are not explained by the standard model. Second, I think we all prefer to see a conventional scientific explanation because it gives more understanding. Time and again the quest for a conventional scientific explanation has triumphed over seemingly impossible obstacles. And finally, I want to ultimately understand that, with all the particle physics one day worked out, life is possible in the universe because π is between 3.14159 and 3.1416. To me, understanding this would be the real anthropic principle. I don't want to lose it.

On the other hand, it is not clear to me which alternative approaches to the cosmological constant are actually worth mentioning in the ten minutes remaining. I will make a slightly eccentric choice, guided by the following considerations.

Supersymmetry, if unbroken, would give a natural explanation of the vanishing of the cosmological constant.¹ It seems a pity to waste it.

Moreover, unbroken supersymmetry, if valid, give a natural explanation of the stability of spacetime. This last point perhaps needs some explanation. The Einstein action $\int d^4x \sqrt{g} R$ has no obvious positivity or stability property. Positive energy of classical General Relativity can nonetheless be proved – for instance, using the possibility of extending the classical theory to include fermions in a supersymmetric way. But positive energy and stability of the vacuum fail in many plausible extension of General Relativity such as nonsupersymmetric Kaluza-Klein theory [7,8]. I think it is a mystery why any nonsupersymmetric string vacuum would be stable. Hence, I wonder if we can somehow make use of the vacua with unbroken supersymmetry to describe nature.

Moreover, a decade ago, one might have hoped that the supersymmetric string vacua were nonperturbatively inconsistent. It has become pretty clear with all the results of the 1990's on nonperturbative string dualities that this is not so. If they are consistent, it is a shame to waste them.

Can we somehow reinterpret the real world in terms of *unbroken* supersymmetry, suitably construed, even though the boson and fermion masses are different?

¹ This is not automatic, since unbroken supersymmetry can coexist with negative cosmological constant, as I have mentioned earlier. But in many situations, with higher dimensions, chiral symmetries, natural microscopic constructions via Calabi-Yau manifolds, etc., unbroken supersymmetry does make vanishing of the cosmological constant natural.

I know of two tries in this direction. Both involve relating the observed $D = 4$ universe to a world of a different dimension. One scenario starts with $D < 4$, and the other starts with $D > 4$.

For the first scenario [9], we consider a supersymmetric string vacuum in $D = 3$ (or similarly in $D < 3$) with the dilaton or string coupling constant g as the only modulus. Despite the supersymmetry of the vacuum, because of a peculiar three-dimensional infrared divergence, the boson and fermion masses are not equal – they are equal at tree level, but are split by gravitational corrections.

We assume that, for example because of a discrete R -symmetry, the cosmological constant vanishes. Now we take $g \rightarrow \infty$. We have learned in the 1990's that sometimes when a coupling constant goes to infinity, a new dimension of spacetime opens up. Let us assume this occurs in the present case. The cosmological constant will remain zero because it is zero for all g . Conceivably, as $g \rightarrow \infty$, the boson and fermion masses (which were unequal for all finite nonzero g) remain unequal. To know, we would need some new insight about dynamics. If so, the $g \rightarrow \infty$ limit would be a four-dimensional world with vanishing cosmological constant and unequal bose-fermi masses. It is not obvious that it could be interpreted, in a dual description, as a four-dimensional world with conventional spontaneously broken supersymmetry, but perhaps this is also possible.

Are there any models that actually exhibit the rather optimistic dynamics that I have just suggested? One reason that it is hard to be find out is that models whose nonperturbative dynamics we know something about usually have many moduli and can undergo partial or complete decompactification even for weak coupling. Because of the way the scenario depends on an infrared divergence that is limited to $D \leq 3$, I think that it is unlikely that the desired dynamics will occur in any model that can undergo partial decompactification to $D \geq 4$ at fixed coupling. This will make it hard to learn in the near future with known methods if the scenario does work as hoped.

Now, I will discuss an alternative idea, based on a suggestion by Gregory, Rubakov, and Sibiryakov [10]. They proposed a scenario in which the world looks four-dimensional up to a very large (presumably cosmological) length scale R , but above that length scale is D -dimensional with $D \geq 5$. A discussion of the model aiming to verify that it has the claimed quasi four-dimensional behavior can be found in [11].

Since the GRS scenario does not obey the usual assumptions of four-dimensional low energy effective field theory, which lead to the cosmological constant problem, we should naturally reexamine the problem in this scenario. (In fact, this was done independently in

a paper [12] that appeared on the hep-th bulletin board the same day this talk was given. In this paper, it is also claimed that the GRS scenario leads to scalar-tensor gravity rather than pure tensor and so is excluded experimentally.)

For our purposes, we will start with a world of $D \geq 5$ with unbroken supersymmetry. For example, to be definite, we will take $D = 5$. We assume that the five-dimensional cosmological constant vanishes. (This is natural in some supergravity models in five dimensions, and would, according to Nahm’s theorem, automatically occur in all supersymmetric models in $D \geq 8$.) We assume that we live on a four-dimensional “brane” which is nonsupersymmetric (that is, non-BPS). This assumption makes the observed macroscopic four-dimensional physics nonsupersymmetric [13]. It is natural for the brane to be flat because this gives a minimal volume hypersurface in five-dimensional Minkowski space.

So we naturally get a flat four-dimensional world with unequal masses for bosons and fermions. But do we observe four-dimensional gravity? The idea of GRS was to get approximate four-dimensional gravity by “localizing” a graviton on the brane along the lines of Randall and Sundrum [14]. The GRS idea can be illustrated with a metric of the general form

$$ds^2 = dr^2 + \left(e^{-2r/r_0} + \epsilon \right) \sum_{i=1}^4 dx_i dx^i. \quad (1)$$

Here $r \geq 0$; there is some sort of “brane” at the $r = 0$ boundary of spacetime. If $\epsilon = 0$, this metric describes a portion of five-dimensional Anti de Sitter space, with negative cosmological constant. We could get more of Anti de Sitter space by continuing to $r = -\infty$. According to the AdS/CFT correspondence, quantum gravity in this five-dimensional spacetime (with $-\infty < r < \infty$) is equivalent to an ordinary four-dimensional conformal field theory, without gravity. The four-dimensional world is the virtual “boundary” at $r = -\infty$. Randall and Sundrum truncated the Anti de Sitter space to $r \geq 0$, and observed that in this case one does get dynamical four-dimensional gravity.² In particular, if $\epsilon = 0$, a four-dimensional graviton is “localized” near $r = 0$.

GRS modified this scenario by taking ϵ nonzero but tiny. As long as $e^{-2r/r_0} \gg \epsilon$, the correction is unimportant. This bound on r means that if the four-dimensional lengths are

² To be more precise, one gets four-dimensional gravity coupled to a cutoff version of the conformal field theory; the low energy physics is conventional four-dimensional physics with General Relativity coupled to some massless matter fields that are governed by a nontrivial infrared fixed point. The cusp-like behavior of the metric for $r = +\infty$ corresponds in the AdS/CFT correspondence to the nontriviality of the IR fixed point.

not too large, the physics will look effectively four-dimensional, just as if $\epsilon = 0$. However, if we take $r \rightarrow \infty$, the ϵ term dominates and the metric becomes five-dimensional Minkowski space. Hence, at very long distances, the world is effectively five-dimensional. (I have parametrized the correction in a way that makes it appear that extreme fine-tuning is required to get ϵ sufficiently small. GRS present the discussion in a way that makes this look more plausible, though their analysis has a “negative energy” problem that I will mention later.)

One apparent problem with this scenario (apart from possible issues raised in [12]) is that it cannot be realized with matter fields that obey even the weakest form of a positive energy condition that is usually considered physically acceptable, which is that the stress tensor T_{ij} obeys $n^i n^j T_{ij} \geq 0$ for any lightlike vector n . Indeed, given this energy condition, one can prove a “holographic c -theorem” in the AdS/CFT correspondence [15-17]. The c -theorem says that as one flows to large r , the effective five-dimensional cosmological constant can only become more negative. Any metric that looks like the ultraviolet end of Anti de Sitter space near $r = 0$, and like five-dimensional Minkowski space near $r = +\infty$, violates this bound. In fact, a general five-dimensional metric with four-dimensional Poincaré symmetry can be put in the form

$$ds^2 = dr^2 + e^{2A(r)} \sum_{i=1}^4 dx_i dx^i, \quad (2)$$

with some function $A(r)$. As shown in section 4.2 of [17], the weak energy condition implies that $A'' \leq 0$. This inequality makes it impossible to have a metric that behaves qualitatively like (1) in the whole range of $r \geq 0$. This point has been made independently in [11].

GRS in fact incorporated a brane of negative tension in their construction of the model. This is one way to violate the weak energy condition. It seems likely that the physics with violation of the weak energy condition is unstable. I do not know if, starting in $D > 5$, it is possible to get a metric of the qualitative form needed for the GRS scenario while also respecting the weak energy condition.

I will conclude by briefly mentioning some other interesting recent developments that were omitted in the actual talk at DM2000 for lack of time. On the one hand, there are recent proposals [18,19] attempting to use a five-dimensional solution with a singularity (as well as a brane on which standard model fields are localized) to get a small cosmological constant. The behavior of these models will depend very much on what assumption is

made about the physics of the singularity. Note that one should not expect that a generic singularity will turn out to have any sensible physical interpretation at all. Elliptic equations (such as the ones that describe time-independent solutions of General Relativity) have an abundance of singular solutions, and only special ones can be given a physical interpretation at the quantum level.

I will make a digression on this point. A familiar example of a singular solution that should *not* be given a physical interpretation is the singular Dirac monopole solution of QED. We do not want to predict magnetic monopoles in QED with a mass that (in units of the electron mass) would only depend on the fine structure constant! By contrast to this example, all of the singular charge-bearing “brane” solutions of ten and eleven-dimensional supergravity have turned out to be approximations to objects that actually exist in the pertinent quantum theories. Apart from the fact that fortune sometimes favors the brave, why did this occur? I think it may be that in the presence of gravity, since a charge-bearing singularity can be hidden behind a black hole horizon, objects carrying all of the gauge charges must exist. This contrasts with QED which does not include gravity and does not have magnetic monopoles. In any event, the generic singularity of a time-independent solution of General Relativity is much “worse” – much less likely to have a quantum interpretation – than the minimal charge-bearing singularities.

So the singularities encountered in [18,19] may well not have any sensible physical interpretation at all, especially in cases in which (as described in the second paper in [19]) the singular solution has a modulus that controls the four-dimensional cosmological constant. If the solution with the singularity were physically sensible, one would expect the modulus to correspond to a physical field, which would then relax to minimize the vacuum energy – not necessarily reaching the value zero.

If a singularity does have a physical interpretation in terms of four-dimensional effective field theory, perhaps with some fields supported near the singularity, this still seems likely at best to lead to a restatement of the problem of the cosmological constant, in a new context. The best hope might be that the framework of low energy effective four-dimensional field theory would somehow break down because of the behavior near the singularity, perhaps leading to something roughly along the lines of GRS.

Finally, another interesting development is the study of nonplanar loop diagrams in noncommutative geometry [20]. A surprising connection between infrared and ultraviolet dynamics was found, wherein what would usually be regarded as the ultraviolet region of a Feynman diagram produces infrared singularities. This evades the usual renormalization

group thinking by means of which one usually deduces an effective low energy field theory description – which leads to trouble with the cosmological constant. (However, the specific phenomenon found in [20] can be described in a low energy effective field theory, albeit one that contains fields that one might not expect.) As has been noted by the authors of [20], such an IR/UV connection might, if other elements fall into place, be an ingredient in an eventual solution of the cosmological constant problem.

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